On Some Properties of Goss Zeta Function

Yujia Qiu

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Abstract

Considered as one of the most important as well as one of the most difficult problems in number theory, Riemann hypothesis is still beyond reach. The *Riemann zeta function* is defined to be the complex function $\zeta(s) := \prod_{p \text{ prime}} (1 - p^{-s})^{-1} = \sum_{n \geq 1} n^{-s}$. It converges for all $s = a + bi$ with the real part a larger than 1. Besides the trivial zeroes at negative even integers, the Riemann hypothesis asserts that the non-trivial zeroes all have real part 1/2. It is known that the distribution of its zeroes has a close relation to that of prime numbers, and its values at negative integers are rational and play an important role in the theory of modular forms. However, the analogue of Riemann hypothesis with respect to Goss zeta function in function field is already proved. As an analogue to classical Riemann zeta function, Goss zeta function is defined for an integral ring A of a global function field of positive characteristic, which can be considered as a map from the ring $\mathbb{Z}_p \times \mathbb{C}_\infty^*$ to the ring of power series over $\mathbb{Z}_p$. In the 90's, Professor Daqing Wan(1996) and Jeffrey Sheats(1998) proved the analogue of Riemann hypothesis for Goss zeta function defined over global function fields corresponding to the projective lines. It has applications in modern cryptography and coding theory, especially in algebraic geometry codes.

My work concerns the partial Goss zeta function, which is defined as a partial sum of the complete Goss zeta function corresponding to fixed equivalent class modulo a prime ideal of the integral ring A. My aim is to develop a similar theory for partial Goss zeta functions. To be precise, the special values of partial zeta functions are all power series. In particular, their special values at negative integers are polynomials. I want to investigate the distribution of zeroes of these special values.

In this talk I will give a brief introduction to the Goss zeta function and some known results. At the end I will state and try to explain to proof to some results from my work.