Lumpy Labor Adjustment as a Propagation Mechanism of Business Cycles

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Motivation

- New generation of RBC models emphasize the role of the labor market in generating business cycle fluctuations.
  - ✓ Search and Matching: Merz (1995) , Andolfatto (1996)
  - ✓ Factor hoarding: Burnside and Eichenbaum (1996)
  - ✓ Habit formation: Wen (1998)
  - ✓ Learning-by-doing: Chang Gomes and Schorfheide (2002)

Motivation

They have two features in common:

- Introducing lagged labor into dynamics.
- Representative agent model.

- Empirical evidence of lumpy, asynchronous labor adjustment.
  - Caballero and Engel (1993)
  - Caballero et al. (1997)
  - Varejao and Portugal (2006)

Motivation

- Empirical evidence of lumpy, asynchronous labor adjustment.
  - Caballero and Engel (1993)
  - Caballero et al. (1997)
  - Varejao and Portugal (2006)
- Information about distribution of sub-units is crucial to linking micro-level features with implications for macro behavior deduced by the aggregation mechanism.
  - Hamermesh (1989, 1993), Hamermesh and Pfann(1996)

What I do

- I pursue the lumpy labor adjustment as a propagation mechanism for business cycles

- In the literature, GE (S,s) models are used to address this question.

  Khan and Thomas (2003)

  King and Thomas (2006)

What I do

- Applying stochastic process to model the lumpy labor adjustment in a DSGE model.
  - Basic model: Poisson process
    - Staggered adjustment (Taylor,1980 and Calvo,1983)
    - Constant Hazard function
  - Extended model: Weibull distribution
    - Increasing Hazard function

Outline:

- Baseline Model
- Weibull Adjustment Model
- Calibration & Results
- Conclusion

Model

- An economy with labor market frictions, which randomly cause some fraction of firms not to reoptimize labor.
- Firms form a common expectation of the faction of non-adjusting firms depending on time. (Hazard function)
- Continuum of firms are differentiated by their stocks of employment.
- Firms are indexed according to their vintage group label j.

$$L_t = \sum_{j=0}^{\infty} \theta(j) l_{j,t}$$

Labor Adjustment Process

- Simplifying the real-world adjustment decisions in terms of generic trails that satisfy:
  - ✓ Each trial has two outcomes: Adjusting or not adjusting
  - ✓ The probability of adjusting is $H(j)$
    - ✓ (i) Poisson case $$H(j) = \frac{\theta(j)}{1 - F(j)} = \frac{1-\alpha}{\alpha}$$
    - ✓ (ii) Weibull case $$H(j) = \frac{k}{\lambda} \left(\frac{j}{\lambda} \right)^{k-1}$$

Time-invariant Distribution

Markov Process :

$$\begin{bmatrix} \theta_{1,t} \\ \theta_{2,t} \\ \vdots \\ \theta_{j,t} \\ \vdots \end{bmatrix} = \begin{bmatrix} \bar{\alpha}_1 & 1-\bar{\alpha}_1 & 0 & \dots & 0 & \dots \\ \bar{\alpha}_2 & 0 & 1-\bar{\alpha}_2 & \dots & 0 & \dots \\ \vdots & \vdots & & \ddots & & \\ \bar{\alpha}_j & 0 & \dots & 0 & 1-\bar{\alpha}_j & \dots \\ \vdots & \vdots & & & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \theta_{1,t-1} \\ \theta_{2,t-1} \\ \vdots \\ \theta_{j,t-1} \\ \vdots \end{bmatrix}$$

Time-invariant distribution of firms across vintages:

$$\Theta, \text{ such that } (I - P')\Theta = 0.$$

Time-invariant Distribution

Poisson Case:

$$\theta(j) = (1-\alpha)\alpha^j$$

Weibull Case:

$$\theta(j) = F(j+1) - F(j) = exp\left(-\left(\frac{j}{\lambda}\right)^k\right) - exp\left(-\left(\frac{j+1}{\lambda}\right)^k\right),$$

Simplified (S,s) model

Firm's problem, given(Θ , Z_t)

$$\max_{l_t, k_{t+j}} \quad V_t = \sum_{j=1}^{\infty} \theta_j E_t\{\tilde{\beta}_{t+j}[F(l_t, k_{t+j}) - W_{t+j} l_t - R_{t+j} k_{t+j}] | \Omega_t\}$$

Cobb-Douglas Production Function:

$$y_{j,t} = F(l_{j,t}, k_{j,t}) = Z_t l_{j,t}^a k_{j,t}^b$$

Technology Shock: $Z_t = \bar{Z}_t z_t,$

where $z_t = z_{t-1}^{\rho} e^{v_t},$ and $v_t \sim i.i.d. N(0; \sigma^2)$

Simplified (S,s) model

HH's Problem:

$$U = \max_{\{C_t, L_t, I_t\}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(U(C_t) - V(L_t) \right) \right\}.$$

s.t.

$$C_t + I_t \leq W_t L_t + R_t K_t + T_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

$$\lim_{T \to \infty} E_0 \left[\prod_{t=0}^{T} R_{t,t+1}^{-1} K_{t+1} \right] = 0,$$

System of Equations

$\sum_{j=0}^{\infty} \theta(j)\, E_t[\tilde{\beta}_{t,t+j}(a z_{t+j} l_{0,t}^{a-1} k_{j,t}^{b} - W_{t+j})] = 0$	Optimal labor demand
$L_t = \sum_{j=0}^{\infty} \theta(j)\, l_{j,t}$	Aggregate labor demand
$R_t = b\, \frac{y_{j,t}}{k_{j,t}} + 1 - \delta$	Capital demand
$K_t = \sum_{j=0}^{\infty} \theta(j) k_{j,t}$	Aggregate capital demand
$\chi L_t^{\phi} C_t^{\eta} = W_t$	Labor supply
$1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\eta} R_{t+1}\right]$	Euler equation
$C_t + K_{t+1} - (1-\delta)K_t = z_t L_t^{a} K_t^{b}$	Budget constraint
$z_t = z_{t-1}^{\rho} e^{v_t}$	Technology shock evolution
$\lim_{T\to\infty} E_0\left[\prod_{t=0}^{T} R_{t,t+1}^{-1} K_{t+1}\right] = 0$	transversality condition

Table 2: Collection of equilibrium equations

Partial Equilibrium

Firm's optimal demand with DRTS (a+b<1)

- Optimal Capital demand

$$R_t = f_k(j,t) = b\ z_t \frac{l_{j,t}^{a}}{k_{j,t}^{1-b}}$$

- Optimal Employment demand

$$l_t^{*\,\frac{1-a-b}{1-b}} = \frac{ab^{b/1-b} \sum_{j=1}^{\infty} \theta_j E_t[\tilde{\beta}_{t+j} z_{t+j}^{1/1-b} / R_{t+j}^{b/1-b}]}{\sum_{i=0}^{\infty} \theta_j E_t[\tilde{\beta}_{t+j} W_{t+j}]},$$

Dynamic labor demand: (CA RBC)

- Firm's level:

$$\frac{(1-a-b)\alpha\beta}{1-\alpha\beta}E_t[\hat{l}_{0,t+1}] - \frac{1-a-b}{1-\alpha\beta}\hat{l}_{0,t} - \frac{b\bar{R}}{\bar{r}}\hat{R}_t - (1-b)\hat{w}_t + z_t = 0$$

- Aggregate level:

$$\alpha\beta\kappa E_t[\hat{l}_{t+1}] - (1+\alpha^2\beta)\kappa\hat{l}_t + \alpha\kappa\hat{l}_{t-1} - \frac{b\bar{R}}{\bar{r}}\hat{R}_t - (1-b)\hat{w}_t + z_t = 0$$

where $\kappa = \frac{(1-a-b)}{(1-\alpha)(1-\alpha\beta)}$

Dynamic labor demand: (WBL RBC)

- Firm's level:

$$\frac{\Psi\beta}{A}(1-a-b)E_t[\hat{l}_{0,t+1}] - \Psi(1-a-b)\hat{l}_{0,t} - \frac{b\bar{R}}{\bar{r}}\hat{R}_t - (1-b)\hat{w}_t + z_t = 0$$

$$\text{Where } \Psi = \sum_{j=0}^{\infty}\theta(j)\beta^j$$

- Aggregate level: $L_t = \theta(0)l_t^* + \theta(1)l_{t-1}^* + \ldots + \theta(J)l_{t-J}^*$

Calibration

Parameters	Values	Implication
β	0.9902	annual real rate 4%
δ	0.025	annual depreciation rate 10%
b	0.329	to match capital to output ratio of 2.35(Thomas and Khan (2004))
a	0.58	labor's share of output (King, Plosser, and Rebelo (1988))
η	1	$logC$ common in the literature
ϕ	0	Indivisible labor assumption (Hansen,1985)
α	0.77	average adjustment rate of 0.23 Caballero and Engel (1993)
λ	1.38	characteristic life of the Weibull distribution
k	1.2	increasing hazard function Varejão and Portugal (2006)
ς	0.95	Solow residual estimate,
σ^2	0.007	Solow residual estimate,

Calibration

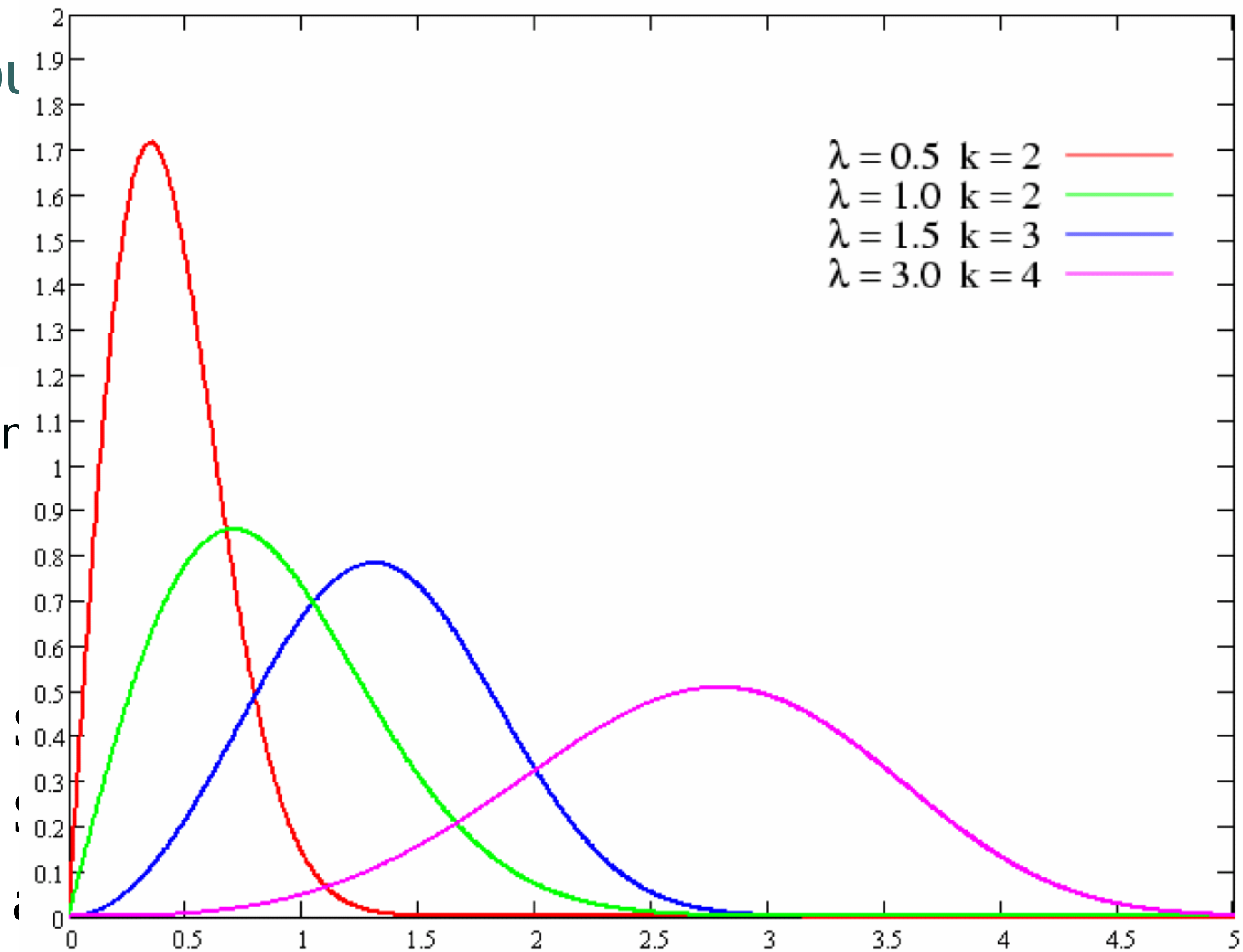
Weibull Distribu

PDF:

Hazard functior

Parameters:

- λ :
- K :
- α :



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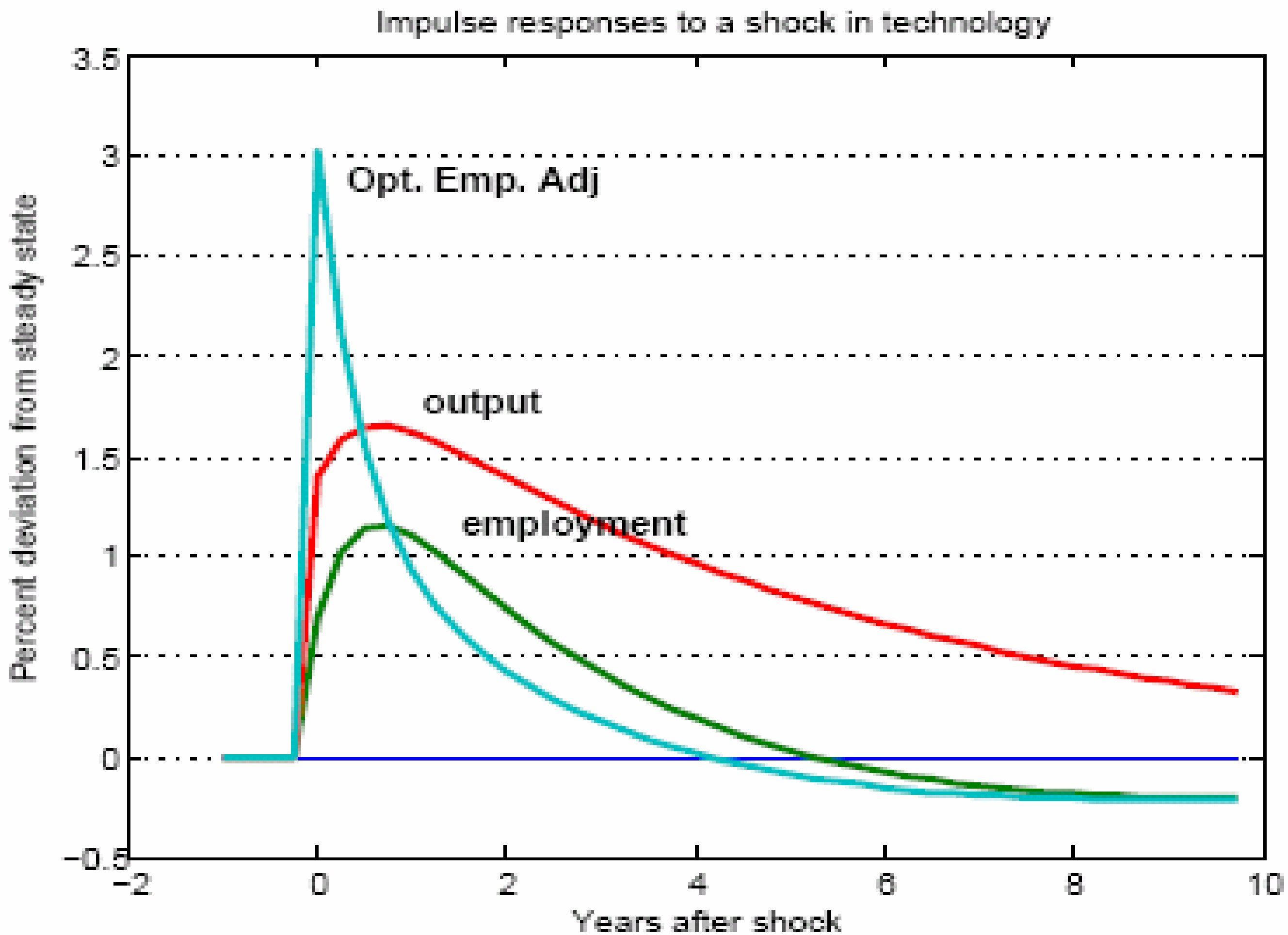
Impulse Response



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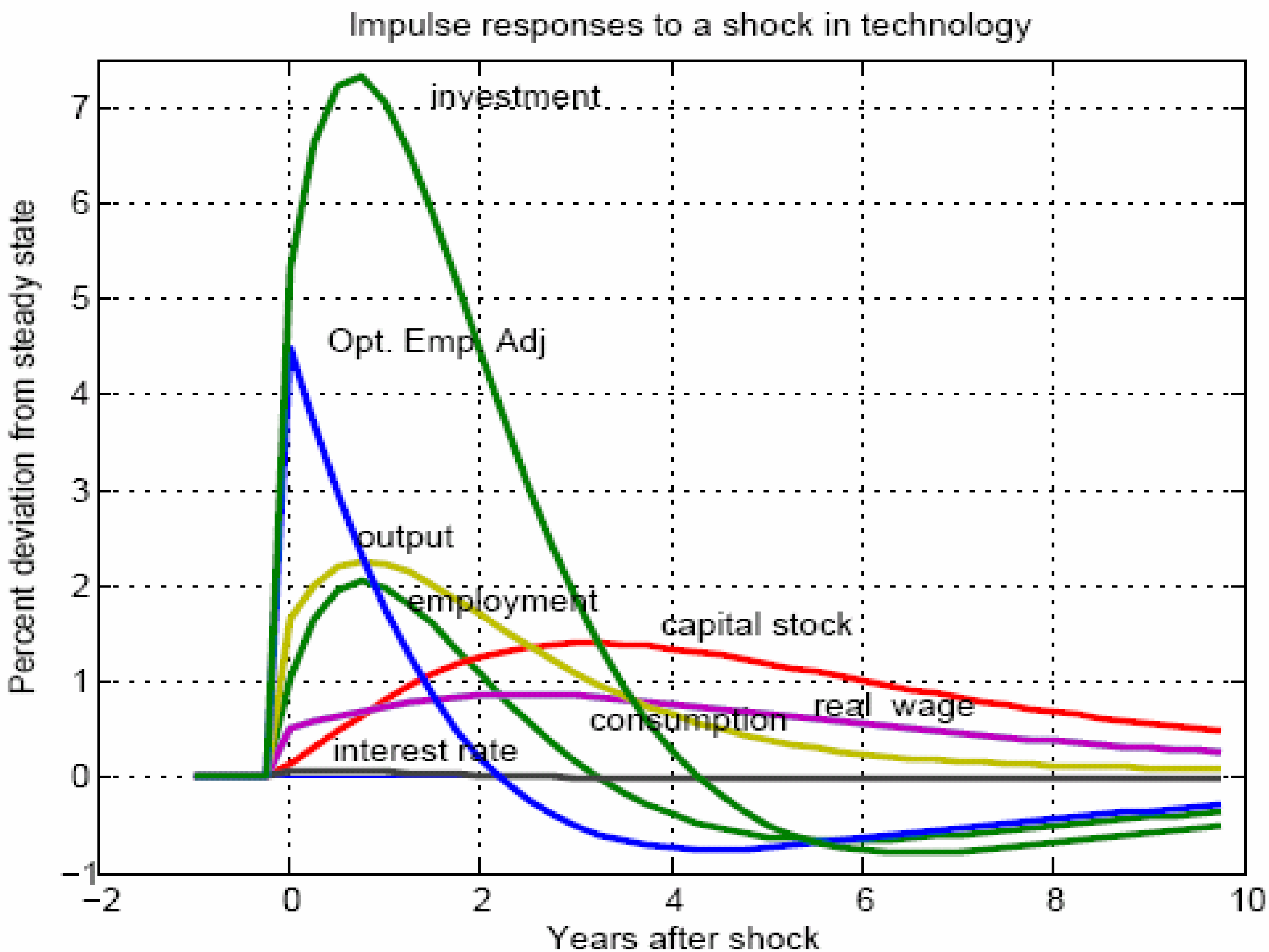

B.C. Implications

Variables	*U.S.Data*[a]	*RBC*	*HF RBC*	*CA RBC*	*WBL RBC*
Hours	1.69 (0.98)	0.59 (0.47)	0.36 (0.33)	0.85 (0.63)	1.59 (0.89)
Employment	1.41 (0.82)			0.85 (0.63)	1.59 (0.89)
Real wage	0.76 (0.44)	0.67 (0.54)	0.76 (0.71)	0.37 (0.27)	0.52 (0.28)
Consumption	1.27 (0.74)	0.38 (0.31)	0.36 (0.33)	0.37 (0.27)	0.52 (0.28)
Output	1.72 (1.00)	1.24 (1.00)	1.08 (1.00)	1.35 (1.00)	1.79 (1.00)
Investment	5.34 (3.10)	3.84 (3.10)	3.24 (3.01)	3.74 (2.77)	6.17 (3.44)
Labor productivity	0.73 (0.42)	0.67 (0.54)	0.76 (0.71)	0.59 (0.44)	0.38 (0.21)

a: all statistics are reported in Cooley (1995) Table(1.1)

Table 9: Volatility of Business Cycles

Main Findings

- The baseline model can generate:
  - Lumpy adjustment at micro level (Front-loading Effect)
  - Smooth, persistent dynamics for aggregate hours
  - Humped-shaped Impulse responses
  - Higher volatility of hours than benchmark models

- Weibull adjustment Model can do the same.
  - Even higher volatility of aggregate hours.

Conclusion

- Lumpy labor adjustment introduces stronger propagation mechanism.
- Aggregation mechanism does matter.
- Further research on the micro-fundation of the Weibull distribution (Adjuatment cost structure)