Mathematics and Banking

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Equity and Mathematics

- Plain-Vanilla Warrant
  - Call or Put
  - Black-Scholes Formular

$$C = S\Phi(d_1) - Ke^{-rT}\Phi(d_2)$$

- Exotic Warrant
  - Digital Warrant
  - How to find the price?

Pricing of Digital Warrant

- Using Black-Scholes Formular

$$P = Qe^{-rT}N(d_2)$$

- Is the pricing optimal?
  - no. The price is very sensitive to volatility
  - we use the following formular

    Price = Lim Call(k) – Call (k + 1/N), for n -> infinity

Volatility Issues

- Historical Volatility
  - Dax Vola-New index published by Deutsche Börse and Goldman-Sachs
- Implied Volatility
- Which one is better?
  - example of car driving
  - example of vola before a big event
  - simplicity of historical volatilty
- Stochastic Volatility
  - better for pricing but difficult to obtain
  - input is implied volatility. So the quality of implied volatility is very important, which could cause a very bad stochastic volatility

Implicated Volatility

- Review of the procedure of calculating implied vola for DAX
  - market data
    - DAX-options on eurex, with maturity Dec08, Jan09, Feb09, Mar09, Jun09, Dec09, Jun10, Dec10, Dec11
    - OTM options, as prices are sensitive to volatility
    - Outlier
  - using BS-forumlar and numerical methods to obtain volatilities for those points
  - Interpolation methods are used to obtain a full surface of volatility.
  - Dividends are not taken into consideration, as DAX is performance index.

Controlling and Mathematics

- Statistics gives a overview of the market and risks in a bank
- Example of turbo-warrant and the market
  - What‘s turbo-warrant
  - How does a bank hedge the position:
    - Using future contract with the nearst maturity, e.g. Dax-Future now has maturity Dec08, Mar09, Jun09
  - If the market is volatile, how much exposure a bank faces?
    - The Jerome Kerviel scandal of SocGen,
    - The collapse of Lehman Brothers
  - Let‘s caluclate the risk for a small bank A

Example of Bank A Part I

- The market is volatile
- The position of turbo warrants with underlying DAX:
  - 10k turbo warrants with ratio 1
  - 10k turbo warrants with ratio 0.1
- We carry out simple statistics (Gap-Analysis)
  - take 100 b-days history of open and close of DAX-furture contract with the nearst maturity
  - determine the stubs and calculate the „Häufigkeiten“.

Example of Bank A II

- The result were
  - 10 days, average -100
  - 60 days, average -50
  - 20 days, average 50
  - 10 days, average 100
- An empirical measure is
  - average -100, p = 0.1
  - average -50, p = 0.6
  - average 50, p = 0.2
  - average 100, p = 0.1
- Then we have our expectations = 20
- Our position is 10k*0.1 + 10k = 11k
- Owing to the linearity of the portfolio, we have our risk of 11k *- 20 = -220k